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Gravitational instabilities in helicity-1 waves propagating through matter in equilibrium

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Abstract

It is shown that the interaction of helicity-1 waves of gravity and matter in a thin slab configuration produces new types of instabilities. Indeed, a spin-2 helicity-1 mode interacts strongly with the shear motion of matter. This mode is unstable above a critical wavelength, $\lambda_c = \sqrt{\pi c^2/2G\rho}$. This should be compared with Jeans wavelength, $\lambda_J = \sqrt{\pi c_s^2/G\rho}$, where c_s is the sound speed. The two instabilities are of course different. For the case analyzed, a plane parallel configuration, Jeans instability appears through a density wave perturbation, the material collapsing into a set of plane-parallel slabs. On the other hand, the helicity-1 wave instability induces a transverse motion in the fluid that tends to shear in the material along the node of the perturbation.

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Acoustic waves are affected by gravitational forces. One of the consequences is Jeans instability [1] (see also, e.g., [2, 3]) produced above a certain critical wavelength. There is an electromagnetic counterpart in plasma density waves. In the limit of large wavelengths their frequency square tends to a finite positive value, and the plasma acquires the plasma frequency, whereas in the Jeans case the frequency square turns negative. This is noted by Weinberg [4] but he does not go further. Plasma density waves can be seen as longitudinal electromagnetic waves which are not possible in vacuum but are present in media with the plasma frequency having the role of a photon mass. By the same token acoustic waves can be identified with longitudinal waves of the gravity field but now, instead, they possess an imaginary mass. Transverse electromagnetic waves, in addition to longitudinal waves, are also modified in a plasma and acquire a mass equal to the plasma frequency. The questions posed in this paper concern waves of the gravity field, or wave metric perturbations, propagating in matter. More specifically, we are interested in the following question: are there waves which get a finite real mass or do they get an imaginary mass leading to instabilities other than Jeans?

Waves of the gravity field can be classified according to their spin and helicity. Here we are interested in spin 2 waves which can have helicities 2, 1 and 0. Helicity-2 waves represent radiative gravitational waves and are simply called gravitational waves. Helicity-1 waves, as well as helicity-0, do not exist in vacuum but can propagate in matter. The propagation of waves of the gravity field through matter has been studied in two main contexts, namely, cosmological and astrophysical. Within the cosmological context these waves appear as perturbations of the background universe [5] (see also [6]) and there is now a growing interest in the primordial gravitational wave background since it seems possible to detect it in the future [7, 8]. Within the astrophysical context they have been studied mainly in relation to stars (see e.g., [9]). Usually one considers these stars as either spherically or axisymmetric. The waves in the star, i.e., perturbed quadrupolar or higher polar modes, appear as convective velocity perturbations and density perturbations which through the stellar boundary match with gravitational waves propagating in vacuum [10, 11]. Still within the astrophysical context, there has been another line of research, which is the one that most concerns us here, analyzing non-dissipative [6, 12, 13] and dissipative propagation of waves through matter [14, 15, 16, 17, 6]. The results obtained so far confine themselves either to the short wavelength limit or to transverse helicity-2 gravitational wave modes (e.g., [12, 16]). In either case the interaction of wave and matter is minute, and usually the wave propagates undisturbed.

In this paper we want to go a step further, and reach the level where the wave-

length, λ, is of the order of or larger than the background curvature, $\sim c/\sqrt{G\rho}$. This is not possible to achieve for spherical or quasi-spherical geometries, since it implies systems with a size of the order of their Schwarzschild radius. However, extended finite thin plates either disk-like or parallelepiped-like are amenable to the large λ regime. In order to progress we will study a specific case, namely a wave propagating through a plane parallel slab. We are then able to find some modes which propagate and truly interact with matter. The transverse helicity-2 gravitational wave mode propagates freely without interaction as has been observed in several papers (see e.g. [6, 13]). On the other hand, a transverse helicity-1 shear wave mode interacts strongly with the shear motion of matter. Indeed, above a certain critical wavelength this mode is unstable, as we will now show.

Consider a fluid with proper energy density ρ, pressure p and with a given equation of state $p(\rho)$. Consider then it has suffered a small departure from a state where it is at rest in a Minkowski space with homogeneous density, ρ_0, and pressure, p_0. The fluid energy-momentum tensor is ($c = 1$)

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu} \; , \tag{1}$$

where u_μ denotes the 4-velocity vector. Greek indices μ, ν, etc., take values 0 (or t), and i, with $i = 1, 2, 3$ (or x, y, z, respectively). The metric $g_{\mu\nu}$ is denoted in the Minkowski limit as $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. The fluid and the gravitational field form a coupled system. The change of $T_{\mu\nu}$ due to a fluctuation in the metric is determined by the conservation laws of energy and momentum, $\nabla^\mu T_{\mu\nu} = 0$ and in turn, the gravitational field is determined by the Einstein equations

$$R_{\mu\nu} = 8\pi G (T_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} T) \; . \tag{2}$$

One has to distinguish between a static gravitational field, $\bar{g}_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu}$, caused if not for anything else by the fluid at rest, from the wave modes $h_{\mu\nu}$ defined as

$$g_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} + h_{\mu\nu} \; . \tag{3}$$

The field $\bar{h}_{\mu\nu}$ is related through Einstein equations,

$$R_{\mu\nu}[\bar{h}] = 8\pi G (\bar{T}_{\mu\nu} - \tfrac{1}{2} \bar{g}_{\mu\nu} \bar{T}) \; , \tag{4}$$

to the energy-momentum tensor, $\bar{T}_{\mu\nu}$, of the fluid in static equilibrium under the external field $\bar{h}_{\mu\nu}$ itself. $\bar{T}_{\mu\nu}$ is characterized by $\bar{\rho}$ and $\bar{p}$, which in turn deviate slightly from ρ_0 and p_0, see below.

We adopt the de Donder gauge (see, e.g., [18, 19, 20]),

$$\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h^\alpha_\alpha = 0\ , \tag{5}$$

for both $h_{\mu\nu}$ and $\bar{h}_{\mu\nu}$. Here and in the following the Minkowski metric $\eta_{\mu\nu}$ is used to raise and lower indices of $h_{\mu\nu}$, $\bar{h}_{\mu\nu}$ and ∂_μ. In this gauge the equations of energy-momentum conservation are in leading order in the gravitational field

$$(\eta^{\mu\alpha} - \bar{h}^{\mu\alpha} - h^{\mu\alpha})\,\partial_\mu T_{\alpha\nu} = \frac{1}{2} T_{\mu\alpha}\,\partial_\nu(\bar{h}^{\mu\alpha} + h^{\mu\alpha})\ . \tag{6}$$

The limit $h_{\mu\nu} = 0$ gives the conservation equations of $\bar{T}_{\mu\nu}$. After subtracting that static contribution one obtains the equations governing the variations $\tau_{\mu\nu} \equiv \delta T_{\mu\nu} \equiv T_{\mu\nu} - \bar{T}_{\mu\nu}$ induced by the fluctuations $h_{\mu\nu}$. In first order

$$\partial^\mu \tau_{\mu\nu} - \frac{1}{2} T^{(0)}_{\mu\alpha}\,\partial_\nu h^{\mu\alpha} = 0\ , \tag{7}$$

where $T^{(0)}_{\mu\nu} = \mathrm{diag}(\rho_0, p_0, p_0, p_0)$ replaces $\bar{T}_{\mu\nu}$, an approximation that will be made whenever $\bar{T}_{\mu\nu}$ is multiplied by the field $h_{\alpha\beta}$ because $\bar{T}_{\mu\nu} - T^{(0)}_{\mu\nu}$ is already suppressed by $\bar{h}_{\alpha\beta}$. Equations (7) give the linearized equations (where, $\dot{} \equiv \partial/\partial t$),

$$\delta\dot{\rho} - (\rho_0 + p_0)\,u_{i,i} + \frac{1}{2}(\rho_0 + p_0)\,\dot{h}_{00} = 0\ , \tag{8}$$

$$(\rho_0 + p_0)\,\dot{u}_i - \delta p_{,i} - \frac{1}{2}(\rho_0 + p_0)\,h_{00,i} = 0\ , \tag{9}$$

which yield $\tau_{\mu\nu}$ as linear functions of $h_{\alpha\beta}$ when considering periodic waves. This system is completed by Einstein equations (2) and (4) resulting in the approximate equality,

$$R_{\mu\nu}[\bar{h} + h] - R_{\mu\nu}[\bar{h}] = 8\pi G \left(\tau_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}(\tau - T^{(0)}_{\alpha\beta}\,h^{\alpha\beta}) - \tfrac{1}{2}T^{(0)} h_{\mu\nu}\right)\ , \tag{10}$$

where the traces are defined as $\tau = \tau_{\mu\nu}\,\eta^{\mu\nu}$ and $T^{(0)} = T^{(0)}_{\mu\nu}\,\eta^{\mu\nu}$. In the right-hand side we neglect terms that go as, omiting the indices, $T\bar{h}h$, $(\bar{T} - T^{(0)})h$ or $\tau\bar{h}$, because they are suppressed by extra factors of $\bar{h}_{\mu\nu}$ and henceforth only appear at G^2 order.

In the weak field limit the left-hand side of Eq. (10) is approximated by $-\frac{1}{2}\Box h_{\mu\nu}$ whereas in the right hand side the explicit dependence of the matter terms in $h_{\mu\nu}$ is neglected. In the study of the propagation of gravitational waves through matter one usually considers the high frequency regime, where the wavenumber k is much greater than the background curvature, i.e., $k^2 \gg 8\pi G\rho$, enabling one to neglect

in the equations the matter density ρ. However, matter effects become important when k^2 approaches the background curvature, i.e., $k^2 \sim 8\pi G\rho$. For such small k higher order terms of the Ricci tensor, such as $\triangle \bar{h}_{00}\, h_{\mu\nu} \simeq 4\pi G\rho\, h_{\mu\nu}$, are of the same order as $k^2 h_{\mu\nu}$ and still linear in $h_{\mu\nu}$. This calls for a higher order expansion of the Ricci tensor in equation (10) up to terms linear in both $h_{\mu\nu}$ and background field $\bar{h}_{\mu\nu}$. Such an expansion includes products of $\bar{h}_{\mu\nu}$ derivatives and $h_{\mu\nu}$ derivatives and products of $\bar{h}_{\mu\nu}$ times second derivatives of $h_{\mu\nu}$ so that translational invariance is lost and monochromatic waves cease to be in general solutions of Einstein equations. This may be circumvented in some cases, such as in the solutions presented below.

Consider a thin wall of fluid lying over the plane $x = 0$ with translational symmetry along the $y-z$ plane. If the pressure is everywhere much smaller than the density the Einstein equations (4) admit in first order a Newtonian solution $\bar{h}_{\mu\nu} = 2\phi(x)\, \delta_{\mu\nu}$ with $\phi'' = 4\pi G\bar{\rho}$. Consider in addition that the fluid is in a nearly incompressible state, i.e., with a practically uniform density $\bar{\rho} \simeq \rho_0$. A solution with reflection symmetry about the plane $x = 0$ possess an acceleration $\phi' = 4\pi G\rho_0 x$ and pressure $\bar{p} = 2\pi G\rho_0^2(\ell^2/4 - x^2)$, $\bar{p}$ vanishing at the boundary planes $x = \pm\ell/2$. The density variation is negligible across the wall if $\Delta\bar{p} \ll c_s^2\rho_0$ for a given speed of sound c_s. That is true if the wall thickness ℓ obeys $\ell \ll \lambda_J$, where $\lambda_J = \sqrt{\pi c_s^2/G\rho}$ is the Jeans wavelength. Note that the non-relativistic condition $\bar{p} \ll \bar{\rho}$ is a weaker condition since it implies $\ell \ll 1/\sqrt{G\rho}$. The magnitude of the potential ϕ at the wall depends upon the longitudinal dimension of the wall, L, because at distances larger than L it decays with the inverse distance to the wall. One obtains $|\phi| \sim G\rho_0\ell L$ at the wall. Therefore the Newtonian condition $\phi \ll 1$ yields $\ell L \ll 1/G\rho$. To summarize, the background we are assuming consists of a fluid with uniform density and small pressure associated with a Newtonian gravity potential. Other details, such as the precise fall off of the pressure at the rim of the wall, are no needed here.

To obtain from Eq. (10) an accurate system of linear equations in $h_{\mu\nu}$ up to order of $G\rho$ it is sufficient to have $\bar{h}_{\mu\nu}$ in leading order of $G\rho$ in the left-hand side of Eqs. (10). There are terms proportional to $\phi'' h_{\mu\nu}$, $\phi' h_{\mu\nu,\alpha}$ and $\phi\, h_{\mu\nu,\alpha\beta}$. For definiteness we consider waves propagating along the z axis, i.e., $\tau_{\mu\nu}$ and $h_{\mu\nu}$ varying as $\exp(-i\omega t + ikz)$. It is conceivable that for large enough wavelengths the terms proportional to ϕ' and ϕ are much smaller than the terms with $\phi'' = 4\pi G\rho_0$ so that solutions of that type may be found by neglecting the ϕ' and ϕ terms. It is not enough however that $k^2\phi$ and $k\phi'$ be much smaller than ϕ''. The three kinds of terms mix in the Bianchi identities and therefore one should not expect that the Einstein equations remain always consistent with each other if some terms are neglected. Nevertheless two modes were found that provide consistent solutions of Einstein

equations with definite wavelength along the z axis. They are h_{12} and $h_{02} - h_{23}$.

After neglecting the ϕ' and ϕ terms (and pressure) Eqs. (10) give for the h_{12} mode

$$(\omega^2 - k^2 - 2\phi'')h_{12} = 16\pi G(\tau_{12} - \tfrac{1}{2}\rho_0 h_{12}) \ , \tag{11}$$

while the fluid equations of motion (8) and (9) yield $\tau_{\mu\nu} = 0$. Hence, the h_{12} mode satisfies $\omega^2 = k^2$ and propagates at the vacuum speed of light without disturbing the fluid. This absence of dispersion is in agreement with various previous results regarding the propagation of transverse gravitational waves in media (see, e.g., [14, 6]).

For the h_{02} and h_{23} mode ($kh_{23} = -\omega h_{02}$ in the gauge (5)) Eqs. (10) give, after neglecting the ϕ' and ϕ terms as before:

$$(\omega^2 - k^2)h_{02} = -16\pi G(\delta T^{02} - \tfrac{1}{2}\rho_0 h_{02}) \ , \tag{12}$$

$$(\omega^2 - k^2)h_{23} = 16\pi G(\delta T^{23} - \tfrac{1}{2}\rho_0 h_{23}) \ , \tag{13}$$

where $\delta T^{02} = \rho_0 u^2$ is the momentum density along the $y-$ direction and the fluid velocity is $u^2 = -u_2 + h_{02}$. The terms proportional to $\rho_0 h_{02}$ and $\rho_0 h_{23}$ are due to the background curvature $G\rho_0$. The fluid equations of motion (8) and (9) yield $u_i = 0 = \delta\rho = \delta p$ hence, $u^2 = h_{02}$ and $\delta T^{23} = 0$. The resulting dispersion relation is

$$\omega^2 = k^2 - 8\pi G\rho_0 \ . \tag{14}$$

In contrast to the h_{12}, this $h_{02} - h_{23}$ mode exhibits an instability at very large wavelengths, $k^2 \leq 8\pi G\rho_0$, that is induced by matter. Most notably, the polarization carries the helicity states ± 1 and not ± 2, in contrast to the gravitational waves that propagate in vacuum. In vacuum, of course, this mode can be gauged away through a coordinate transformation but that is no longer true in matter as can be seen by the following argument: h_{23} can be eliminated with a transformation of the type $y \to y + \epsilon(t, z)$ but h_{02} transforms into $h'_{02} = (1 - \omega^2/k^2)h_{02}$, which is not zero, because the phase velocity ω/k is different from unit in matter. Alternatively, one may gauge away u^2 and h_{02} but not h_{23}. It is easy to prove that no coordinate transformation can bring simultaneously all the components $h_{\mu\nu}$ and u^i into zero.

The motion of the fluid can be characterized in a gauge independent way by the covariant derivative of the velocity, $u_{\mu;\nu}$ (see [21]). In leading order, the important non-zero covariant derivatives are

$$u_{2;3} = u_{3;2} = -\frac{1}{2}(\partial_3 h_{02} - \partial_0 h_{23}) \ . \tag{15}$$

It means that this is a shear motion, $u_{\mu;\nu} = \sigma_{\mu\nu}$, with the square of the shear magnitude defined as the gauge invariant quantity $\sigma^2 = \sigma_{\mu\nu}\sigma^{\mu\nu}/2$. In the gauge we use,

$$\sigma = u_{2;3} = -\frac{4\pi G\rho_0}{k^2}\,\partial_3 h_{02}\ , \tag{16}$$

which shows again the crucial role played by matter. These helicity-1 waves can then be called shear waves. The components u^2, h_{02}, h_{23} correspond to what are called vector perturbations in cosmological perturbation theory [22]. They are interpreted as a relativistic generalization of purely rotational fluid flow. Here, they appear exclusively associated to pure shear motion. The vorticity for this mode is zero, since its relevant covariant velocity component $u_2 = 0$ (indeed, from Euler equations: $u_2 = -u^2 + h_{02} = 0$).

We can now discuss the two main aspects of this problem: the first aspect concerns the physics of the instability, and the second the generation and propagation of stable waves.

Starting with the first aspect, one notices from equation (14), that the instability sets in through an $\omega = 0$ mode, with a critical wave number $k_c = \sqrt{8\pi G\rho_0/c^2}$, where the speed of light c has been restored. In terms of wavelength, above the critical value $\lambda_c = 2\pi/k_c$, i.e., for

$$\lambda \geq \left(\frac{\pi c^2}{2G\rho_0}\right)^{1/2}\ , \tag{17}$$

the system is unstable, $\lambda = \lambda_c$ being the marginal case. This instability tends to shear in the material along the node of the perturbation. Indeed, under the effect of the wave the particles of the fluid get an acceleration $\Gamma_{200} = \dot{h}_{02}$ that produces the velocity $u^2 = h_{02}$. As shown in Figure 1, one sees that part of the material moves to one side, part to the other, whereas it stays at rest at the node where the velocity in the $y-$direction, u^2, is zero.

A very important point which should be made explicitly clear is that at such critical wavelengths there are equilibrium configurations which are not at all dynamically unstable. Indeed, in a previous paragraph we have put the following limits on the equilibrium configuration, $\ell \ll c_s/\sqrt{G\rho}$ and $\ell L \ll c^2/G\rho$. To have $L > \lambda_c \sim c/\sqrt{G\rho}$ while the above conditions are still satisfied, ℓ has to satisfy both limits $\ell \ll c_s\lambda_c/c$ and $\ell \ll \lambda_c^2/L$. Thus, for sufficiently small thickness ℓ, there are well defined equilibria.

In order to better understand such an instability it is advisable to rearrange equation (17) above and write it as $\mu \geq \pi m_\ell/\lambda$, where $\mu \equiv \sigma\lambda \equiv \rho_0\ell\lambda$ is the mass

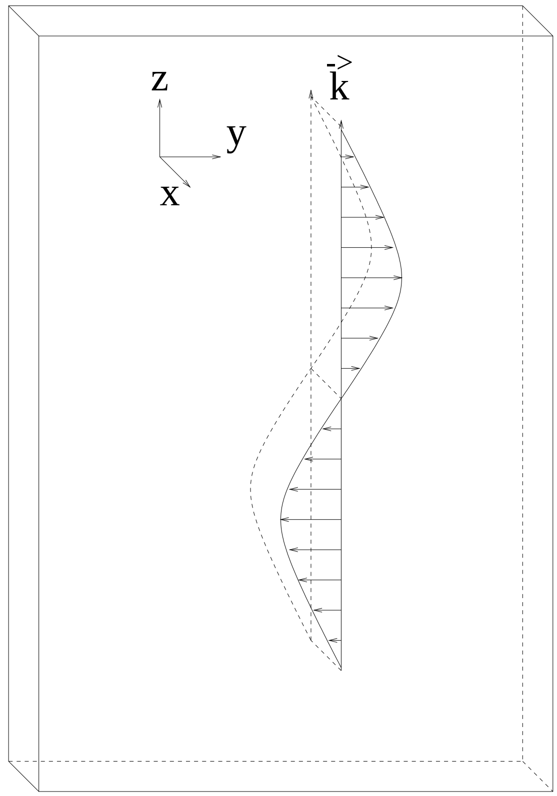

Figure 1: A helicity-1 shear wave of gravity travels in the $z-$direction through a plane parallel slab and induces a motion on the fluid in the $y-$direction. The static gravitational potential is a function of the x coordinate.

per unit length in the $y-$direction and $m_\ell \equiv c^2\ell/2G$ is the reference relativistic mass associated with the thickness length ℓ. Thus m_ℓ/λ can be seen as the relativistic reference mass per unit length in the $z-$direction. The instability criterion (17) says that when the mass per unit length in the $y-$direction of the perturbation is about greater than the reference mass per unit length in the $z-$direction then the momentum in the $y-$direction is enough to overcome the pull back from the mass along z and the material breaks apart.

This instability is of course different from Jeans instability but not quite in what regards the gravitational field dynamics. Equation (12) can be written as $\Box h_{02} = 8\pi G\, \delta T^{02}$, because $u^2 = h_{02}$, and if one performs a Lorentz boost along the $y-$direction, it is equivalent to $\Box h'_{00} = -8\pi G\, \delta T'^{00}$ in the new frame, which is comparable to the equation governing the response of the gravitational field to density fluctuations. What is different is that they are not accompanied by perturbations

in the fluid proper density and pressure as in the Jeans, acoustic case, but rather by transverse displacements of the fluid. For such a plane layer and density wave perturbations of the form $\exp(-i\omega t + ikz)$, Jeans instability sets in when $k \leq k_J$ with $k_J = \sqrt{4\pi G\rho_0/c_s^2}$, or $\lambda \geq \lambda_J$ with $\lambda_J = \sqrt{\pi {c_s}^2/G\rho_0}$. The material collapses into a set of plane-parallel slabs distributed perpendicularly to the $z-$direction and uniformly along the $z-$ direction [23, 24].

The instability we have found here sets in through the interaction of shear waves and matter. A helicity-1 shear wave travelling through the plane parallel layer in the $z-$direction with $\lambda \geq \lambda_c$ will induce an instability in the matter and in the gravitational field itself. For a planar slab characterized by a surface density σ and height ℓ one can write the critical wavelength as $\lambda_c = \sqrt{\pi c^2 \ell/2G\sigma}$. One sees that λ_c can be relatively small for high enough surface densities or very thin systems. Conversely, if such instabilities are not detectable within the system, one can put a lower limit on the ratio ℓ/σ; for a slab with size L these instabilities do not show up if $\lambda_c \gtrsim L$, i.e., $\ell/\sigma \gtrsim (2G/\pi c^2)L^2$. An estimate of λ_c for our Galaxy yields $\lambda_c \simeq 16\,000\,\mathrm{Kpc}$ showing that it is not thin enough to present such instabilities. However, for extended and very thin systems this instability is applicable and could set in.

Within plane parallel symmetric configurations, other unstable modes should occur, although we have been able to solve the system consistently only for the $h_{02} - h_{23}$ mode because we have restricted this study to plane waves. It would be interesting to find a consistent solution for the mode involving h_{00}.

We have considered here plane parallel symmetry but other type of configurations, such as disk-like, should also possess such instabilities. For disk-like configurations some other features and new global instabilities could set in. Jeans instabilities show surprises in the disk-like case, since for high enough values of the sound speed a disk is stable to all axisymmetric perturbations [25, 26], a feature that does not appear in the spherical and planar cases. Domain walls and cosmic strings should be considered. In a preliminary calculation, we have found that domain walls also present this type of instability to wave perturbations of the gravity and scalar field [27]. On the other hand, the spherical case is less interesting since here $\lambda_c \sim R^{3/2}/R_s^{1/2}$, where R_s is the Schwarzschild radius of an object of radius R. Thus, the instability sets in when $R \sim R_s \sim \lambda_c$, and other stronger dynamical instabilities must already be present.

We now discuss the second aspect, i.e., the generation and propagation of stable waves of helicity-1. In order to estimate the energy flux of these waves we use the expression for the effective stress-energy $t_{\mu\nu}$ tensor given in [18] and apply to our

case. It yields

$$t_{\mu\nu} = \frac{1}{16\pi G} < -h_{02,\mu}h_{02,\nu} + h_{23,\mu}h_{23,\nu} >= -\frac{\rho_0}{2k^2} < h_{02,\mu}h_{02,\nu} > , \tag{18}$$

where the symbol $< >$ denotes an average over one wave period. Defining $u^2 = |u| \exp(-i\omega t + ikz)$, with $|u|$ being the amplitude of the velocity in the $y-$direction, we find

$$t^{03} = \frac{1}{2}\rho_0 |u|^2 v , \tag{19}$$

where v is the velocity of propagation of the wave. It is now difficult to estimate at this point a realistic energy flux, given one has to deal with the propagation as well as the generation of such a wave. One possibility is to have a source of mass M in an oscillatory (orbital) movement of radius R, inducing tidal motions on particles at a distance r. This induces a tidal velocity to the particles which we identify with $|u|$, given then by $|u|/c \sim \sqrt{R_s R}/r$, where $R_s = 2GM/c^2$. If we put $R_s \sim 10^{-6}R$, $R \sim 10^{-2}r$ and $\rho_0 = 1\,\mathrm{g/cm^3}$ we obtain $|u|/c \sim 10^{-5}$ and, for $v \sim c$, an energy flux $t^{03} \sim 10^{20}\,\mathrm{erg/(s\,cm^2)}$. Perhaps, the most important point is to notice that this tidal perturbation propagates away through the matter at typically the speed of light according to the dispersion relation (14) and properties we have derived for this helicity-1 wave. One can then think that randomly distributed waves of the type considered, should induce thermal motions in the matter, which would be detected as velocity dispersions in the stars or particles of the system. These velocities, in turn, would give the impression of the existence of an extra Newtonian gravitational potential which, for instance, could be interpreted as coming from dark matter, or from modifications of the gravitational potential itself.

The propagation of these helicity-1 shear waves and the instabilities themselves we have been studying here can be considered wave detectors in potential and deserve close attention.

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